AS759 - Homework #2

Due: Thursday, October 10, 2013 at start of class

Note: You are welcome to use tables of integrals and/or symbolic integrators when solving all problems. Just be sure to note down the generic form of the integral if you solve a problem in this way. Potentially helpful formulae are given on the last page.

1. **(15 points)** At the present day, the number density of galaxies in the universe, $\phi(L)dL$, with luminosities between L and $L + dL$ is given by the Schechter luminosity function:

$$\phi(L)dL = n_* \left(\frac{L}{L_*}\right)^\alpha \exp\left(-\frac{L}{L_*}\right)\frac{dL}{L_*},$$

where $n_* = 0.019h^3\text{Mpc}^{-3}$, $L_* = 8.0 \times 10^9 h^{-2} L_\odot$, and $\alpha = -0.7$ in the R-band.

a) Use Schechter's luminosity function to compute the total R-band luminosity density of the universe at the present day.

b) Consider a flat (i.e., critical density) Newtonian universe. Recall that on HW#1 we showed that the mass density of such a universe the present day is given by:

$$\rho_{c,0} = \frac{3H_0^2}{8\pi G}$$

Compute the average R-band mass-to-light ratio that the universe would have to have in order to be a flat, Newtonian universe. Express your final answer in units of $M_\odot/L_\odot$, and be sure to hold on to your factors of h throughout.

2. **(20 points)** Consider a flat, matter dominated universe (i.e., a universe in which matter is the only important component of the energy density). At a time, t_e, in the past (corresponding to a redshift z) a galaxy emits a photon which is received at a detector today (t_0).

a) Compute the explicit dependence of the expansion factor on time, $a(t)$. Express $a(t)$ in terms of a_0, t_0 and t **only**. Hint: to eliminate H_0 from your expression, recall the relationship between t_0 and H_0 for this type of universe.

b) Compute the comoving distance to a galaxy in this universe as a function of its redshift.

c) Compute the lookback time to a galaxy in this universe as a function of its redshift and PLOT a graph of $\Delta t(z)$ for $0 \le z \le 5$. (Note: your answer for Δt will depend on H_0.) Comment on the magnitude of Δt for $0 \le z \le 1$ and $4 \le z \le 5$.

d) If $H_0 = 70$ km/s/Mpc, what is the proper radial distance to the particle horizon at the present day?

3. **(20 points)** Once again, consider a flat, matter-dominated universe. Feel free to refer to answers from Question #2 in order to answer this question.

a) Compute the mass in solar units ($M_{\odot}$) within the particle horizon, M_H, as a function of time and as a function of redshift.
b) Suppose we can only see out to a maximum redshift z_*. How much mass is there today (i.e. $t = t_0$) in the volume of the universe out to z_*?
c) Refering to part b), how much mass is there within the particle horizon at $t = t_*$ (i.e. the time corresponding to z_*)? Compare this value to the value in part b).

4. **(10 points)** Compute the redshift, z, at which the angular diameter, δ, of a galaxy with linear diameter, D, reaches a minimum in a flat, matter-dominated universe.

5. **(20 points)** Repeat Question 2 for a flat, radiation-dominated universe (i.e., a universe in which the only important component of the energy denisty is radiation).

6. **(20 points)** Consider a radiation-dominated universe with a present value of the density parameter of $\Omega_0 = 0.25$, a present value of the Hubble parameter of $H_0 = 70$ km/s/Mpc.

a) How old is this universe today (i.e. $t = t_0$)?
b) If the oldest stars in this universe formed at a redshift $z_* = 10$, how old are the oldest stars in this universe?
c) What is the value of the density parameter, Ω_*, when $z = z_* = 10$?

7. **(20 points)** At one time, a popular model for the universe was the "Steady-State Theory". In a Steady-State universe, space is flat and it is assumed that both the Hubble parameter and the average density of matter in the universe are constant in time (i.e. $H(t) = H(t_0) = H_0$ and $n(t) = n(t_0) = n_0$, where t_0 is the age of the universe and n is the matter density). Perform the following calculations for the Steady-State universe.

a) What is the lookback time to a galaxy as a function of its redshift?

b) Compute the present-day proper radial distance to a galaxy (i.e. $R_g(t_0) = a_0\Theta_g$) as a function of z.

c) Show that the proper radial distance to the event horizon is given by $R_E(t) = a(t)\Theta_E(t) = c/H_0$.

d) Consider a comoving volume of the universe (i.e. the volume of the universe out to the comoving coordinate Θ). Suppose that the volume today is $V(t_0) = V_0$ and that there are a total of N_0 galaxies within it. In units of N_0, how how many particles were in the comoving volume one Hubble age (H_0^{-1}) ago?

e) How does the number of galaxies inside the event horizon vary with time?

f) There is something which intuitively you should find to be very silly about the Steady-State model. What is it?

Potentially Useful Formulas

$$H = \frac{1}{a}\frac{da}{dt} \qquad a = \frac{a_0}{(1+z)} \qquad \Omega = \frac{8\pi G\rho}{3H^2} \qquad q = \frac{1}{H^2}\left(-\frac{1}{a}\frac{d^2a}{dt^2}\right)$$

$$H^2 = \frac{8\pi G\rho}{3} - \frac{kc^2}{a^2} \quad \text{(no exotic energy)}$$

$$\left(\frac{H}{H_0}\right)^2 = \Omega_{r0}(1+z)^4 + \Omega_{m0}(1+z)^3 + (1-\Omega_0)(1+z)^2 \quad \text{(no exotic energy)}$$

$$d_A = \frac{c}{H_0 q_0^2(1+z)^2}\left[zq_0 + (q_0-1)(\sqrt{2q_0z+1}-1)\right]$$

$$H_0\Delta t = \int_0^z \left(\frac{H_0}{H}\right)\frac{dz'}{1+z'}$$

$$\Theta_g = \int_0^{r_g} \frac{dr}{(1-kr^2)^{1/2}} = \int_{t_e}^{t_0} \frac{c\,dt}{a(t)} \quad \text{(comoving distance to a galaxy)}$$

$$\Theta_H(t) = \int_0^t \frac{c\,dt'}{a(t')} \qquad \Theta_E(t) = \int_t^\infty \frac{c\,dt'}{a(t')}$$