AS759 - Homework #1
Due: Tuesday, Sept. 24 at the start of class

Note: You are welcome to use tables of integrals and/or symbolic integrators when solving problems. Just be sure to note down the generic form of the integral if you solve a problem in this way.

1. **(10 points)** Prior to the establishment of the extragalactic nebulae as galaxies in their own right, it was supposed by some astronomers in the early 20th century that M31 (the Andromeda nebula) was a coalescing star surrounded by a disk of gas, perhaps destined to become a planetary system.

a) Suppose that the gaseous disk is roughly 160 AU in diameter (i.e., about twice the diameter of the solar system). Given that M31's disk subtends an angle of about 3 degrees, what would the trigonometric parallax of M31 have been? *Express your final answer in units of arcseconds.*

b) Compute the resolving power of the human eye for optical wavelengths ($\lambda = 500$ nm), assuming that the diameter of the pupil of the eye is 0.3 cm. *Express your final answer in units of arcseconds.* How does your answer to part b) compare to your answer to part a)? (Compute a ratio.) Comment on why should the astronomers of the time have suspected that M31 really wouldn't have been a coalescing star, based upon its expected parallax. (Note: the first measurement of a genuine stellar parallax was made in 1838 for the star 61 Cygni, which has a parallax of about 0.3 arcsec.)

c) The greatest accuracy with which trigonometric parallaxes have been made to date is about 0.002 arcsec. If M31 had a parallax that was "just measurable" by today's standards, what would the physical size of its disk be in AU? Comment on the likelihood of such a disk collapsing to form a single star.

2. **(10 points)** von Maanen claimed to have measured proper motions in the spiral nebula known as M101 which amounted to 0.02 arcseconds per year. If von Maanen's observations were correct, and given that M101 is a face–on disk galaxy with a circumference of about 0.5°, at what velocity would its disk have to have been rotating if the disk's physical radius were comparable to the size Shapley claimed for our own Galaxy (i.e., a radius of about 45 kpc)? Compare your result to the velocity of light. In comparison to Shapley's size for the the Milky Way, how big would the disk of M101 have to have been if its alleged proper motions were actually caused by rotation at a relatively "sensible" velocity of 200 km/s? How would this have helped to shore up Shapley's argument that the spiral nebulae were objects which were contained within our own Galaxy?

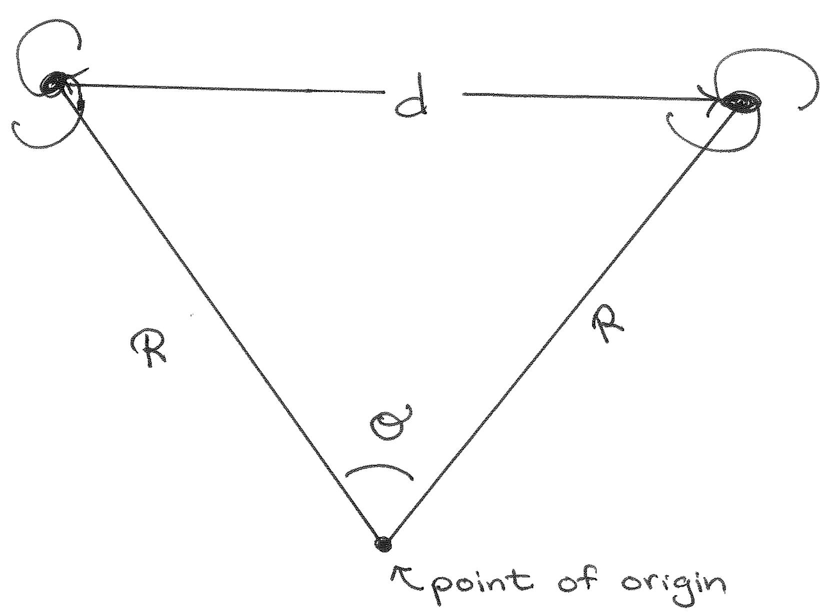
3. **(15 points)** In class we considered a spherical balloon that was expanding uniformly, and we showed that on the surface of that balloon the separation between any two locations increases according to Hubble's law ($v = \frac{dd}{dt} = \frac{1}{a}\frac{da}{dt}d = Hd$, where a is the expansion factor. In this problem we will ask the following question: "Suppose that all galaxies originated simultaneously from a single point and are following radial trajectories through space. What, then, would we observe to be the rate of change of distance between any two galaxies?"

Let's start the problem at some time $t_i \neq 0$ when the galaxies are a distance R_i from the central point. As we did in class, let us allow the radial coordinate to evolve as $R(t) = R_i\, a(t)$, where $a(t)$ is simply the time dependence of the radial distance for all galaxies. Assume that the geometry of the problem is Euclidean (i.e., flat space).

a) Compute the distance, d, between any two galaxies. Express your final answer in terms of $R(t)$ and θ, where θ is the angle between the galaxies (see diagram). Hint: recall the cosine rule from elementary geometry.

b) Compute the rate of change of distance, $\frac{dd}{dt}$, between the galaxies. Express your answer in terms of the expansion factor, a, and d (i.e., eliminate all dependence on R from your equation). How does your result compare to the result that we derived in class for the motion on the surface of the balloon?

c) Discuss how this contrived scenario could be distinguished *observationally* from a uniformly expanding universe. To do this, consider what you would observe if you lived on one of the galaxies and studied the distribution of galaxies along: [i] a radius that points towards the "origin", [ii] a radius that points away from the "origin", and [iii] along a random direction in the universe.



4. **(20 points)** When we discussed Hubble's Law we considered the relative motions of metal stickers pasted to the surface of a spherical balloon which expanded uniformly. Let us now reconsider this analogy and imagine a spherical balloon of radius R which is being inflated over some extended period of time. If at $t = 0$ the balloon had a radius of R_0 then after some length of time, t, the balloon will have expanded by an amount $a(t)$ and its radius at time t will therefore be given by $R(t) = R_0 a(t)$. Imagine also that there are metal stickers pasted on the surface of the balloon and let s be the distance between 2 of the stickers (as measured along the surface of the balloon). In class we showed that the distance between any 2 stickers increases as $v = Hs$ where $H = \frac{1}{a}\frac{da}{dt}$.

a) Imagine that ants reside on each of the metal stickers and that an ant living on one of the stickers wants to visit a friend living on one of the other stickers. The ant steps off his sticker and crawls in a straight path toward the sticker on which his friend resides. The ant travels along the surface of the balloon at some finite constant speed, v'. Show that if the distance between the stickers increases faster than the ant crawls (i.e. $v = \frac{ds}{dt} > v'$) the maximum distance the ant can crawl is:

$$s_{\max} = \frac{av'}{\frac{da}{dt}}$$

This is the distance to the ant's "horizon" and if at any time the distance between the ant's sticker and his friend's is larger than $s_{\max}$, the ant will never reach his friend!

b) Compare the rates at which the distance to the ant's horizon increase for two scenarios: (i) $\frac{da}{dt}$ = constant (i.e. there is no deceleration in the ant's expanding universe) and (ii) $\frac{da}{dt}$ decreases with time (i.e. the expansion rate decelerates). **Comment briefly on your results.**

c) Finally, suppose that H happens to be a constant with a value of 70 km/s/Mpc. Compute the length of time it would take for the balloon to expand sufficiently from its initial size such that the distance between any two metal stickers increases by a factor of 2. Express your final answer in years. **Would you say that a universe with a present–day value of 70 km/s/Mpc for Hubble's constant is expanding rapidly or slowly (on a "human" time scale)?**

5) **(10 points)** In a Newtonian Universe space is flat and the only important component of energy is matter. In addition, kinetic energy in the Newtonian Universe exactly balances potential energy (i.e., $KE = |PE|$). Consider a galaxy located at a distance r from the center of a Newtonian Universe that expanding according to Hubble's law. Show that the mean density of this universe at the present day is given by:

$$\langle \rho_0 \rangle = \frac{3H_0^2}{8\pi G}$$

and compute the value of $\langle \rho_0 \rangle$ in the case that $H_0 = 70$ km/s/Mpc. Express your final answer in units of g cm^{-3} and $M_\odot$ Mpc^{-3}

6) **(20 points)** Consider a Newtonian Universe that is expanding according to Hubble's law. This is a universe that is geometrically flat and matter-dominated. Further assume that there are no large clumps of mass in the universe so that you may treat the universe as having a smooth, uniform density.

Due to the overall expansion of the universe, the distance between any two galaxies moving with the pure Hubble flow scales as $R(t) = a(t)R'$, where R' is the distance between the galaxies at some fiducial time, t'. Assume that the total number of galaxies in the universe does not evolve with time and approximate a typical galaxy as a sphere of radius 10 kpc. If the average number density of galaxies at the present day is 0.005 Mpc^{-3} and $H_0 = 70$ km/s/Mpc, at what redshift is the probability 50% that any line of sight will intersect a typical galaxy?

Hints: Recall that the expansion factor at any redshift is related to its present day value through $a = \frac{a_0}{(1+z)}$ and that $H = \frac{1}{a}\frac{da}{dt}$, so $H = \frac{-1}{(1+z)}\frac{dz}{dt}$. Also note that the expression for ρ in Question 5 above is actually valid at any epoch, so that $\langle \rho(z) \rangle = \frac{3H(z)^2}{8\pi G}$. Recall as well that $dP = n\sigma\, dl = n\sigma c\, dt$, where n is the number density of galaxies and σ is the cross-section.